

Single Limiting Factor Analysis

Q1. The managers of Albion plc are reviewing the operations of the company with a view to making operational decisions for the next month. Details of some of the products manufactured by the company are given below.

Product	AR2	GL3	HT4
Selling price (£/unit)	21.00	28.50	27.30
Material R2 (kg/unit)	2.0	3.0	3.0
Material R3 (kg/unit)	2.0	2.2	1.6
Direct labour (hours/unit)	0.6	1.2	1.5
Variable production overheads (£/unit)	1.10	1.30	1.10
Fixed production overheads (£/unit)	1.50	1.60	1.70
Expected demand for next month (units)	950	1,000	900

Products AR2, GL3 and HT4 are sold to customers of Albion plc. Albion plc manufactures a wide range of products in addition to those detailed above.

Material R2, which is not used in any other of Albion's products, is expected to be in short supply in the next month because of industrial action at a major producer of the material. Albion plc has just received a delivery of 5,500 kg of Material R2 and this is expected to be the amount held in stock at the start of the next month. The company does not expect to be able to obtain further supplies of Material R2 unless it pays a premium price. The normal market price is £2.50 per kg.

Material R3 is available at a price of £2.00 per kg and Albion plc does not expect any problems in securing supplies of this material. Direct labour is paid at a rate of £4.00 per hour.

Required:

- Calculate contribution margin per unit of AR2, GL3 and HT4
- Determine the optimum production schedule for Products AR2, GL3 and HT4 for the next month, on the assumption that additional supplies of Material R2 are not purchased.
- If Albion plc decides to purchase further supplies of Material R2 to meet demand for Products AR2, GL3 and HT4, what should be the maximum price per kg that the company is prepared to pay?

Q2. Shilton Ltd produces three chemicals X, Y and Z. The selling price and cost per litre for each of these products are budgeted as follows:

	X	Y	Z
	£/litre	£/litre	£/litre
Selling price	100	120	120
Direct materials	20	16	21
Direct labour (£12 per hour)	18	24	27
Other direct expenses	–	3	–
Variable overhead	12	16	18
Fixed overhead	6	8	9

Notes

- The fixed overhead is absorbed on the basis of labour hours, based on a budget of 440 hours per month.
- Maximum demand for each product for month 4 is as follows:
 - X 150 litres
 - Y 40 litres
 - Z 60 litres
- Included in the maximum demand totals is an unavoidable commitment to a major customer to supply 15 litres per month of each of the three products.
- During month 4 there is a shortage of labour hours that will restrict production. The total number of labour hours available is 375 hours.
- Shilton is able to produce and sell fractions of a litre.

Required:

Determine the production mix that will maximise profit in month 4 and calculate the resulting profit. (12 marks)

Q3. A Company is currently preparing its budget for the year ending 30th September 19X2. The company manufactures and sells three products, X, Y and Z.

The unit selling price and cost structure of each product is budgeted as follows.

	X	Y	Z
	£	£	£
Selling price	100	124	32
Variable cost:			
Labour	24	48	06
Material	26	07	08
Overhead	10	05	06
	60	60	20
Contribution per unit	40	64	12

Direct labour rate is budgeted at £6/hour, and fixed costs at £1,300,000 per annum. The company has a maximum production capacity of 228,000 direct labour hours

A meeting of the board of directors has been convened to discuss the budget and to resolve the problem as to the quantity of each product, which should be made and sold. The sales director presented the results of a recent market survey, which reveals that market demand for the company's product will be as follows

Product	Units
X	24,000
Y	12,000
Z	60,000

The production director proposes that since Z only contributes £12 per unit, the product should no longer be produced, and the surplus capacity transferred to produce additional quantities of X and Y. The sales director doesn't agree with the proposal.

Z is considered necessary to complement the product range and to maintain customer goodwill. If Z is not offered, the sales director believes that sales of X and Y will be seriously affected. After further discussion the board decided that a minimum of 10,000 units of each product should be produced. The remaining production capacity would then be allocated so as to achieve the maximum profit possible.

Required: -

Prepare a budgeted statement which clearly shows the maximum profit which could be achieved in the year ending 30th September 19X2 after considering the decision taken by the company.

Q4. A company uses limiting factor analysis to calculate an optimal production plan given a scarce resource.

The following applies to the three products of the company:

Product	I	II	III
	£	£	£
Direct materials (at £6/kg)	36	24	15
Direct labour (at £10/hour)	40	25	10
Variable overheads (£2/hour)	8	5	2
Total variable cost	84	54	27
Maximum demand (units)	2,000	4,000	4,000
Optimal production plan	2,000	1,500	4,000

How many kg of material were available for use in production?

A 15,750 kg

B 28,000 kg

C 30,000 kg

D 38,000 kg

Q5. A company makes three products for which details per unit are given.

	Product X	Product Y	Product Z
Selling price	£18	£35	£50
Direct material	£4	£5	£5
Direct labour hours	0.5	2	2.5

The direct labour rate is \$8.00 per hour. Direct labour hours are limited.

In which order should the products be ranked to achieve the maximum profit with the available labour hours?

	First	Next	Last
A	X	Y	Z
B	Y	Z	X
C	Z	X	Y
D	X	Z	Y

Q6. A company makes three products for which the following details are given.

	Product P (\$)	Product Q (\$)	Product R (\$)
Selling price per unit	20	24	36
Direct material per unit	9	12	15
Direct labour per unit	5	3	9

The same material is used by all three products and it costs \$3.00 per kilo. There is a shortage of material. In which order of priority should the products be made in order to achieve maximum profit from the available material?

	first	next	last
A	P	Q	R
B	Q	R	P
C	R	P	Q
D	R	Q	P

Q7. The table contains information for the two products of a company.

Product	X	Y
Contribution per unit	\$12	\$9
Machine hours required per unit	6	3
Estimated sales demand	200	200
Required machine hours	1200	600

Machine capacity limited to 1200 hours. What is the maximum possible contribution?

A \$2100	B \$3000	C \$3300	D \$4200
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Q9. A company manufactures Product A. For its production, company manufactures three components, alpha, beta and gamma who are then assembled (in the ratio of 1:1:1) to produce product A. The budget for the next year calls for the production and assembly of 4,000 of product A.

The variable production cost per unit of the final product, product A, is as follows:

	Machine hour	Variable cost (£)
1 unit of alpha	3	20
1 units of beta	4	36
1 unit of gamma	4	24

Only 26,000 hours of machine time will be available during the year, and sub-contractor has quoted the following unit prices for supplying components:

Alpha £29; Beta £40; gamma £34.

Required: -

Compute production level that company could achieve for each component and amount of components it should purchase in order to fulfill requirement of product A.

Q10. Mabbutt plc makes four types of electrical sub-assembly: A, B, C and D. Demand and costs per unit in the coming period are estimated to be as follows:

Sub-assembly	A	B	C	D
Demand (units)	10,000	50,000	120,000	60,000
Unit variable costs	£ per unit	£ per unit	£ per unit	£ per unit
Direct labour (£4.00 per hour)	4	6	8	6
Direct material	2	5	2	11
Variable overhead	2	3	4	3
Total variable cost per unit	8	14	14	20
Absorbed fixed overhead	8	12	16	12
Total production cost	16	26	30	32

Fixed overheads are absorbed on the basis of direct labour hours and represent apportioned general factory overhead. The direct labour used to build the sub-assemblies is highly skilled and Mabbutt plc sometimes has difficulties in recruitment, resulting in shortages of labour. Due to rapid technological change, stocks of completed sub-assemblies are never carried.

The finance director of Mabbutt plc considers the costs of the sub-assemblies to be too high and is considering subcontracting their manufacture. A supplier has offered to supply any quantity of A, B, C or D for £10, £16, £13 and £25 per unit respectively.

Mabbutt plc seeks to satisfy demand at minimum cost.

Required:

(a) On a purely financial basis, determine how many of each sub-assembly should be made by Mabbutt plc and how many should be bought in from the supplier, if Mabbutt plc expects direct labour hours in the coming period to be:

(i) in unlimited supply;

(4 marks)

(ii) restricted to 145,000 hours.

(8 marks)

Linear Programming

Q1. A profit-seeking firm has two constraints: labour, limited to 16,000 hours, and materials, limited to 15,000kg. The firm manufactures and sells two products, X and Y. To make X, the firm uses 3kg of material and four hours of labour, whereas to make Y, the firm uses 5kg of material and four hours of labour. The contributions made by each product are \$30 for X and \$40 for Y. The cost of materials is normally \$8 per kg and the labour rate is \$10 per hour.

Required: –

By using linear programming, calculate production plan which will maximize profit.

Q2. Dauntless Ltd aims to maximise its profits from the two products (X and Y) which it manufactures and sells. The selling prices per unit for products X and Y are £220 and £206 respectively. At these prices the company can sell all that it can produce. The following product cost data is available:

	Product X £/unit	Product Y £/unit
Material L (£6 per litre)	30	36
Material M (£7.50 per litre)	45	30
Other variable costs	55	44

In the first three months of next year the supply of material L will be limited to 24,000 litres. However in the second three month period both material L and material M will be in short supply and each will be limited to 24,000 litres. The company holds no stocks.

Required:

(a) Determine the optimal production plan in units for the first three months of next year and the resultant total contribution. (4 marks)

The company's management accountant has already carried out some preliminary calculations relating to the second three month period. Using linear programming, she has determined that the optimal production plan for that quarter involves a combination of product X and product Y.

(b) Determine the optimal production plan in units for the second three month period of next year and the resultant total contribution. (6 marks)

Q3. A company is currently manufacturing two types of batteries, Reliance star and Reliance super star. Following are details related to its production

	Assembly Department	Testing Department
Reliance star	2.0 hours	1.5 hours
Reliance super star	4.0 hours	1.0 hours

The assembly department has maximum of 12,000 hours available per month. The testing department has a maximum of 6,000 hours available per month. Because of limited availability of component parts for the reliance star batteries, the maximum demand of reliance star batteries per month is 3,500.

Summerised financial data about each product are given as follows

	Reliance star	Reliance super star
Selling price	£2,200	£3,500
Direct material	£700	£1100

Variable assembly department costs are £300 per hour. Variable testing department costs are £200 per hour.

Required: –

What is the optimal mix of the reliance star and reliance super star product per month in order to maximise profit.

Shadow price

- Shadow price of a product is increase in value (usually extra contribution) which would be created by having available one additional unit of limiting factor.
- It therefore represents maximum premium that the firm would be willing to pay for one extra unit of constraint.
- If extra cost of any proposal is less than the shadow price then it is affordable by the business and should accept the offer.
- Non-critical constraints (or slack) will have zero shadow price as they are already available

Q4. Higgins Co (HC) manufactures and sells pool cues and snooker cues. The cues both use the same type of good quality wood (ash) which can be difficult to source in sufficient quantity. The supply of ash is restricted to 5,400 kg per period. Ash costs \$40 per kg.

The cues are made by skilled craftsmen (highly skilled labour) who are well known for their workmanship. The skilled craftsmen take years to train and are difficult to recruit. HC's craftsmen are generally only able to work for 12,000 hours in a period. The craftsmen are paid \$18 per hour.

HC sells the cues to a large market. Demand for the cues is strong, and in any period, up to 15,000 pool cues and 12,000 snooker cues could be sold. The selling price for pool cues is \$41 and the selling price for snooker cues is \$69.

Manufacturing details for the two products are as follows:

	Pool cues	Snooker cues
Craftsmen time per cue	0.5 hours	0.75 hours
Ash per cue	270 g	270 g
Other variable costs per cue	\$1.20	\$4.70

HC does not keep inventory.

Required:

(a) Calculate the contribution earned from each cue. (2 marks)

(b) Determine the optimal production plan for a typical period assuming that HC is seeking to maximise the contribution earned. You should use a linear programming graph (using the graph paper provided), identify the feasible region and the optimal point and accurately calculate the maximum contribution that could be earned using whichever equations you need. (12 marks)

Some of the craftsmen have offered to work overtime, provided that they are paid double time for the extra hours over the contracted 12,000 hours. HC has estimated that up to 1,200 hours per period could be gained in this way.

Required:

(c) Explain the meaning of a shadow price (dual price) and calculate the shadow price of both the labour (craftsmen) and the materials (ash). (5 marks)

(d) Advise HC whether to accept the craftsmens' initial offer of working overtime, discussing the rate of pay requested, the quantity of hours and one other factor that HC should consider. (6 marks)

Q5. The Cosmetic Co is a company producing a variety of cosmetic creams and lotions. The creams and lotions are sold to a variety of retailers at a price of \$23.20 for each jar of face cream and \$16.80 for each bottle of body lotion. Each of the products has a variety of ingredients, with the key ones being silk powder, silk amino acids and aloe vera. Six months ago, silk worms were attacked by disease causing a huge reduction in the availability of silk powder and silk amino acids. The Cosmetic Co had to dramatically reduce production and make part of its workforce, which it had trained over a number of years, redundant.

The company now wants to increase production again by ensuring that it uses the limited ingredients available to maximise profits by selling the optimum mix of creams and lotions. Due to the redundancies made earlier in the year, supply of skilled labour is now limited in the short-term to 160 hours (9,600 minutes) per week, although unskilled labour is unlimited. The purchasing manager is confident that they can obtain 5,000 grams of silk powder and 1,600 grams of silk amino acids per week. All other ingredients are unlimited. The following information is available for the two products:

	<u>Cream</u>	<u>Lotion</u>
Materials required:		
-- silk powder (at \$2.20 per gram)	3 grams	2 grams
- silk amino acids (at \$0.80 per gram)	1 gram	0.5 grams
- aloe vera (at \$1.40 per gram)	4 grams	2 grams
Labour required:		
-- skilled (\$12 per hour)	4 minutes	5 minutes
- unskilled (at \$8 per hour)	3 minutes	1.5 minutes

Each jar of cream sold generates a contribution of \$9 per unit, whilst each bottle of lotion generates a contribution of \$8 per unit. The maximum demand for lotions is 2,000 bottles per week, although demand for creams is unlimited. Fixed costs total \$1,800 per week. The company does not keep inventory although if a product is partially complete at the end of one week, its production will be completed in the following week.

Required:

(a) On the graph paper provided, use linear programming to calculate the optimum number of each product that the Cosmetic Co should make per week, assuming that it wishes to maximise contribution. Calculate the total contribution per week for the new production plan. All workings MUST be rounded to 2 decimal places. (14 marks)

(b) Calculate the shadow price for silk powder and the slack for silk amino acids. All workings MUST be rounded to 2 decimal places. (6 marks)

Q6. Cut and Stitch (CS) make two types of suits using skilled tailors (labour) and a delicate and unique fabric (material).

Both the tailors and the fabric are in short supply and so the accountant at CS has correctly produced a linear programming model to help decide the optimal production mix.

The model is as follows:

Variables:

Let W = the number of work suits produced

Let L = the number of lounge suits produced

Constraints

Tailors' time: $7W + 5L \leq 3,500$ (hours) – this is line T on the diagram

Fabric: $2W + 2L \leq 1,200$ (metres) – this is line F on the diagram

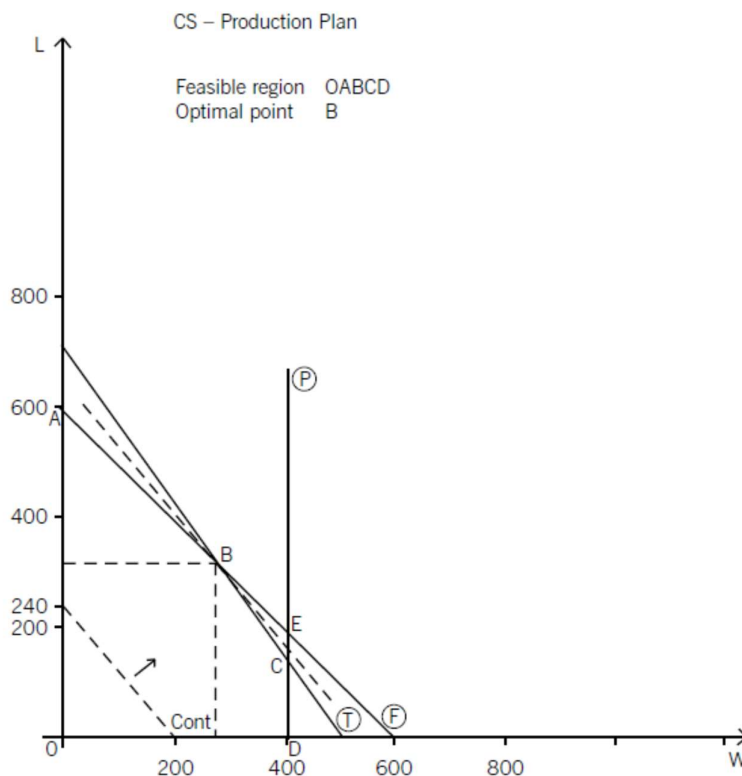
Production of work suits: $W \leq 400$ – this is line P on the diagram

Objective

Objective is to maximise contribution subject to:

$$C = 48W + 40L$$

On the diagram provided the accountant has correctly identified OABCD as the feasible region and point B as the optimal point.



Required:

(a) Find by appropriate calculation the optimal production mix and related maximum contribution that could be earned by CS. (4 marks)

(b) Calculate the shadow prices of the fabric per metre and the tailor time per hour. (6 marks)

The tailors have offered to work an extra 500 hours provided that they are paid three times their normal rate of \$1.50 per hour at \$4.50 per hour.

Required:

(c) Briefly discuss whether CS should accept the offer of overtime at three times the normal rate. (6 marks)

(d) Calculate the new optimum production plan if maximum demand for W falls to 200 units. (4 marks)

(20 marks)

The following scenario relates to questions 21–25

Cara Co makes two products, the Seebach and the Herdorf.

To make a unit of each product the following resources are required:

	Seebach	Herdorf
Materials (\$100 per kg)	5 kg	7 kg
Labour hours (\$45 per hour)	2 hours	3 hours
Machine hours (\$60 per hour)	3 hours	2 hours

Fixed overheads are \$300,000 each month.

The contribution per unit made on each product is as follows:

	Seebach	Herdorf
Contribution (\$ per unit)	250	315

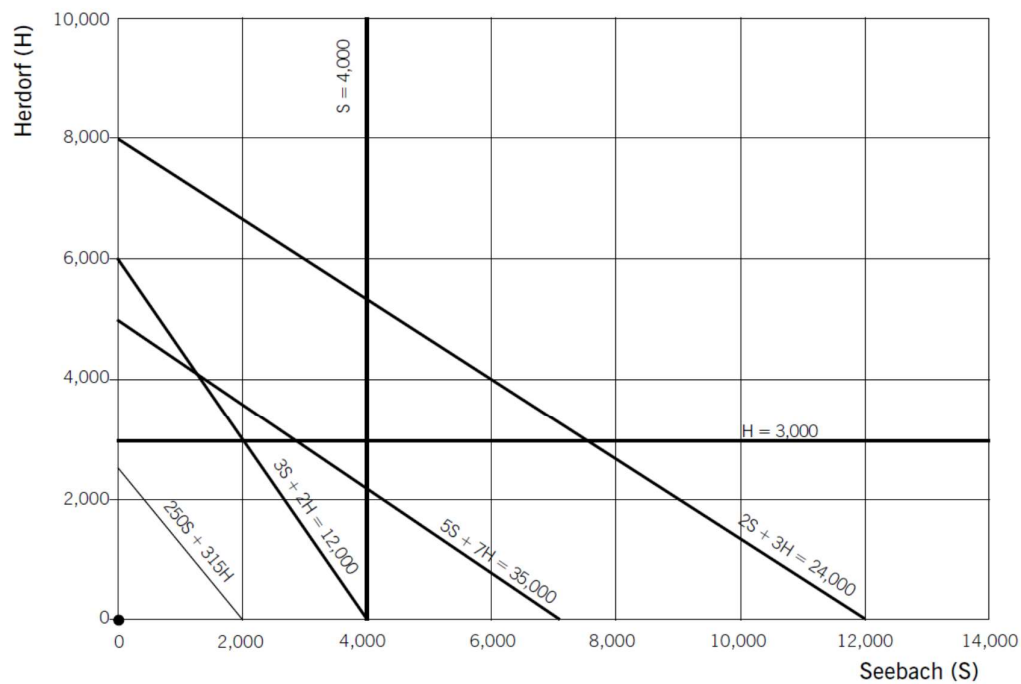
The maximum demand each month is 4,000 units of Seebach and 3,000 units of Herdorf. The products and materials are perishable and inventories of raw materials or finished goods cannot be stored.

Cara Co has a legally binding obligation to produce a minimum of 2,000 units of Herdorf in each of months 1 and 2. There is no minimum production required in month 3.

The manufacturing manager is planning production volumes and the maximum availability of resources for months 1, 2 and 3 are as follows:

Month	1	2	3
Materials (kg)	34,000	42,000	35,000
Labour (hours)	18,000	12,000	24,000
Machine (hours)	18,000	19,000	12,000

For month 3 the following linear programming graph has been produced:



21 What is/are the limiting factor(s) in month 1?

- A Materials, labour hours and machine hours
- B Materials and machine hours only
- C Materials only
- D Labour hours only

22 The production manager has identified that the only limiting factor in month 2 is labour hours.

What is the production volume for Herdorf for month 2 (to the nearest whole unit)?

- A 0
- B 1,333
- C 2,000
- D 3,000

23 If the shadow price for month 2 is \$125 per labour hour, which of the following statements is/are correct?

- (1) The production manager would be willing to pay existing staff a maximum overtime premium of \$125 per hour for the next 2,000 hours
- (2) The production manager would be willing to pay a maximum of \$170 per hour for an additional 2,000 hours of temporary staff time

- A 1 only
- B 2 only
- C Both 1 and 2
- D Neither 1 nor 2

24 What is the maximum profit which can be earned in month 3?

- A \$1,080,000
- B \$1,380,000
- C \$1,445,000
- D \$1,145,000

25 Which of the following interpretations of the linear programming graph produced for month 3 is/are correct?

- (1) Even if demand for either product increases, labour will be a slack variable if no other resources change
- (2) If more machine hours were made available in month 3, they would be used initially to make Herdorfs

- A 1 only
- B 2 only
- C Both 1 and 2
- D Neither 1 nor 2

CSC Co is a health food company producing and selling three types of high-energy products: cakes, shakes and cookies, to gyms and health food shops. Shakes are the newest of the three products and were first launched three months ago. Each of the three products has two special ingredients, sourced from a remote part the world. The first of these, Singa, is a super-energising rare type of caffeine. The second, Betta, is derived from an unusual plant believed to have miraculous health benefits.

CSC Co's projected manufacture costs and selling prices for the three products are as follows:

	Cakes	Cookies	Shakes
Per unit	\$	\$	\$
Selling price	5.40	4.90	6.00
Costs:			
Ingredients: Singa (\$1.20 per gram)	0.30	0.60	1.20
Ingredients: Betta (\$1.50 per gram)	0.75	0.30	1.50
Other ingredients	0.25	0.45	0.90
Labour (\$10 per hour)	1.00	1.20	0.80
Variable overheads	0.50	0.60	0.40
Contribution	2.60	1.75	1.20

For each of the three products, the expected demand for the next month is 11,200 cakes, 9,800 cookies and 2,500 shakes.

The total fixed costs for the next month are \$3,000.

CSC Co has just found out that the supply of Betta is going to be limited to 12,000 grams next month. Prior to this, CSC Co had signed a contract with a leading chain of gyms, Encompass Health, to supply it with 5,000 shakes each month, at a discounted price of \$5.80 per shake, starting immediately. The order for the 5,000 shakes is not included in the expected demand levels above.

Required:

- (a) Assuming that CSC Co keeps to its agreement with Encompass Health, calculate the shortage of Betta, the resulting optimum production plan and the total profit for next month. (6 marks)

One month later, the supply of Betta is still limited and CSC Co is considering whether it should breach its contract with Encompass Health so that it can optimise its profits.

Required:

- (b) Discuss whether CSC Co should breach the agreement with Encompass Health.

Note: No further calculations are required.

(4 marks)

Several months later, the demand for both cakes and cookies has increased significantly to 20,000 and 15,000 units per month respectively. However, CSC Co has lost the contract with Encompass Health and, after suffering from further shortages of supply of Betta, Singa and of its labour force, CSC Co has decided to stop making shakes at all. CSC Co now needs to use linear programming to work out the optimum production plan for cakes and cookies for the coming month. The variable 'x' is being used to represent cakes and the variable 'y' to represent cookies.

The following constraints have been formulated and a graph representing the new production problem has been drawn:

$$\text{Singa: } 0.25x + 0.5y \leq 12,000$$

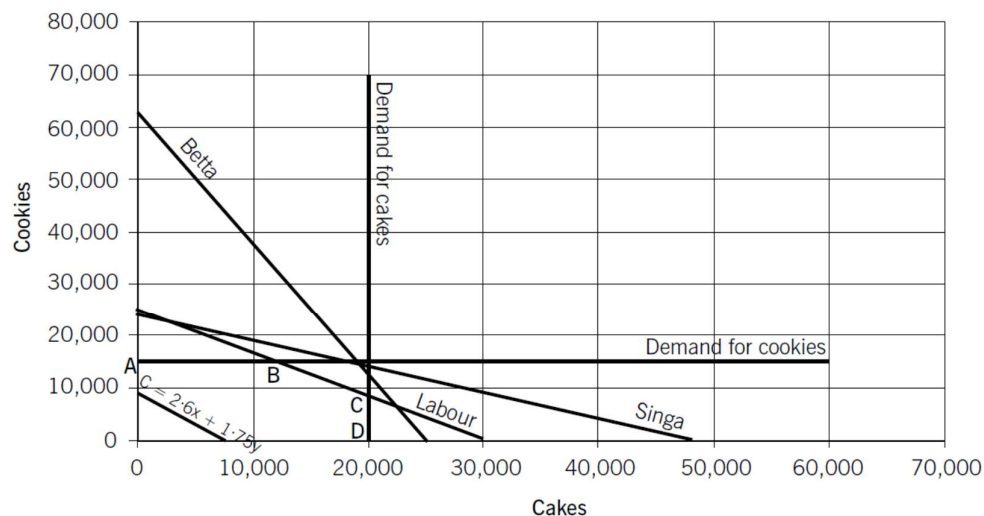
$$\text{Betta: } 0.5x + 0.2y \leq 12,500$$

$$\text{Labour: } 0.1x + 0.12y \leq 3,000$$

$$x \leq 20,000$$

$$y \leq 15,000$$

$$x, y \geq 0$$



Required:

- (c) (i) Explain what the line labelled ' $C = 2.6x + 1.75y$ ' on the graph is and what the area represented by the points OABCD means. (4 marks)
- (ii) Explain how the optimum production plan will be found using the line labelled ' $C = 2.6x + 1.75y$ ' and identify the optimum point from the graph. (2 marks)
- (iii) Explain what a slack value is and identify, from the graph, where slack will occur as a result of the optimum production plan. (4 marks)

Note: No calculations are needed for part (c).

(20 marks)